Zadatak 101 (Mario, gimnazija)

Ako je $x^{-1}+y^{-1}=1,\ x\cdot y=2$ tada je x + y jednako:

$$A.\ 0 \qquad B.\ 1 \qquad C.\ 2 \qquad D.\ 3$$

Rješenje 101

Ponovimo!

$$a^{-n}=\frac{1}{a^{n}} \quad , \quad a^{1}=a \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}=1 \Rightarrow a=b.$$

$$\left.\begin{array}{l} x^{-1}+y^{-1}=1 \\ x\cdot y=2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{1}{x}+\frac{1}{y}=1 \\ x\cdot y=2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{y+x}{x\cdot y}=1 \\ x\cdot y=2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{x+y}{x\cdot y}=1 \\ x\cdot y=2 \end{array}\right\} \Rightarrow \frac{x+y}{2}=1 \Rightarrow x+y=2.$$

Odgovor je pod C.

Vježba 101

Ako je $x^{-1}+y^{-1}=1,\ x\cdot y=3$ tada je x + y jednako:

$$A.\ 0 \qquad B.\ 1 \qquad C.\ 2 \qquad D.\ 3$$

Rezultat: D.

Zadatak 102 (Nino, srednja škola)

Gordana je zbrojila duljine triju stranica pravokutnika i dobila 44 cm. Branka je zbrojila duljine triju stranica istog pravokutnika i dobila 40 cm. Koliki je opseg toga pravokutnika.

$$A.\ 42\ cm \qquad B.\ 56\ cm \qquad C.\ 60\ cm \qquad D.\ 68\ cm$$

Rješenje 102

Ponovimo!

Četverokut je dio ravnine omeđen sa četiri dužine. Konveksni četverokuti su četverokuti kojima su svi kutovi manji od 180°.

Paralelogrami su četverokuti kojima su po dvije nasuprotne stranice usporedne (paralelne).

Pravokutnik je paralelogram koji ima barem jedan pravi kut (pravi kut ima 90°).

Opseg pravokutnika

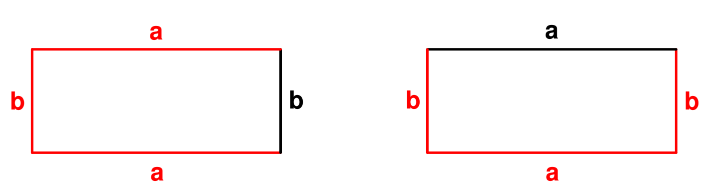
Opseg je zbroj duljina svih stranica pravokutnika

$$O=2\cdot(a+b).$$

$$b\cdot\frac{a}{b}=a \quad , \quad a\cdot\frac{b}{c}=\frac{a\cdot b}{c} \quad , \quad \left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow \left.\begin{array}{l} a+c=b+d \\ a-c=b-d \end{array}\right\}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$



Neka su a i b duljine stranica pravokutnika.

1.inačica

$$\left.\begin{array}{l} 2\cdot a+b=44 \\ a+2\cdot b=40 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow \left.\begin{array}{l} 2\cdot a+b=44 \\ a+2\cdot b=40 \ /\cdot(-2) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{rcases} 2\cdot a+b=44 \\ -2\cdot a-4\cdot b=-80 \end{rcases} \Rightarrow -3\cdot b=-36 \Rightarrow -3\cdot b=-36 \;/:(-3) \Rightarrow b=12\ cm.$$

Računamo duljinu stranice a.

$$\begin{rcases} a+2\cdot b=40 \\ b=12 \end{rcases} \Rightarrow a+2\cdot 12=40 \Rightarrow a+24=40 \Rightarrow a=40-24 \Rightarrow a=16\ cm.$$

Opseg pravokutnika iznosi:

$$\begin{rcases} a=16\ cm \\ b=12\ cm \end{rcases} \Rightarrow \left[\, O=2\cdot(a+b) \,\right] \Rightarrow O=2\cdot(16\ cm+12\ cm) \Rightarrow O=56\ cm.$$

Odgovor je pod B.

2.inačica

$$\begin{rcases} 2\cdot a+b=44 \\ a+2\cdot b=40 \end{rcases} \Rightarrow \begin{bmatrix} \text{zbrojimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow 2\cdot a+b+a+2\cdot b=44+40 \Rightarrow 3\cdot a+3\cdot b=84 \Rightarrow$$

$$\Rightarrow 3\cdot(a+b)=84 \Rightarrow 3\cdot(a+b)=84 \;/\cdot\frac{2}{3} \Rightarrow 2\cdot(a+b)=56 \Rightarrow \left[\, O=2\cdot(a+b) \,\right] \Rightarrow O=56\ cm.$$

Odgovor je pod B.

3.inačica

$$\begin{rcases} 2\cdot a+b=44 \\ a+2\cdot b=40 \end{rcases} \Rightarrow \begin{bmatrix} \text{oduzmemo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow 2\cdot a+b-(a+2\cdot b)=44-40 \Rightarrow 2\cdot a+b-a-2\cdot b=4 \Rightarrow$$

$$\Rightarrow a-b=4.$$

Riješimo sustav jednadžbi.

$$\begin{rcases} a+2\cdot b=40 \\ a-b=4 \end{rcases} \Rightarrow \begin{bmatrix} \text{metoda suprotnih} \\ \text{koeficijenata} \end{bmatrix} \Rightarrow \begin{rcases} a+2\cdot b=40 \\ a-b=4 \;/\cdot(-1) \end{rcases} \Rightarrow \begin{rcases} a+2\cdot b=40 \\ -a+b=-4 \end{rcases} \Rightarrow$$

$$\Rightarrow 3\cdot b=36 \Rightarrow 3\cdot b=36 \;/:3 \Rightarrow b=12\ cm.$$

Računamo duljinu stranice a.

$$\begin{rcases} a-b=4 \\ b=12 \end{rcases} \Rightarrow a-12=4 \Rightarrow a=4+12 \Rightarrow a=16\ cm.$$

Opseg pravokutnika iznosi:

$$\begin{rcases} a=16\ cm \\ b=12\ cm \end{rcases} \Rightarrow \left[\, O=2\cdot(a+b) \,\right] \Rightarrow O=2\cdot(16\ cm+12\ cm) \Rightarrow O=56\ cm.$$

Odgovor je pod B.

4.inačica

$$\begin{rcases} 2\cdot a+b=44 \\ a+2\cdot b=40 \end{rcases} \Rightarrow \begin{rcases} 2\cdot a+b=44 \;/+b \\ a+2\cdot b=40 \;/+a \end{rcases} \Rightarrow \begin{rcases} 2\cdot a+b+b=44+b \\ a+2\cdot b+a=40+a \end{rcases} \Rightarrow \begin{rcases} 2\cdot a+2\cdot b=44+b \\ 2\cdot a+2\cdot b=40+a \end{rcases} \Rightarrow$$

$$\Rightarrow \begin{rcases} 2\cdot(a+b)=44+b \\ 2\cdot(a+b)=40+a \end{rcases} \Rightarrow \begin{bmatrix} \text{zbrojimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow 2\cdot(a+b)+2\cdot(a+b)=44+b+40+a \Rightarrow$$

$$\Rightarrow 4\cdot(a+b)=84+(a+b) \Rightarrow 4\cdot(a+b)-(a+b)=84 \Rightarrow 3\cdot(a+b)=84 \Rightarrow$$

$$\Rightarrow 3\cdot(a+b)=84 \;/\cdot\frac{2}{3} \Rightarrow 2\cdot(a+b)=56 \Rightarrow \left[\, O=2\cdot(a+b) \,\right] \Rightarrow O=56\ cm.$$

Odgovor je pod B.

Vježba 102

Gordana je zbrojila duljine triju stranica pravokutnika i dobila 16 cm. Branka je zbrojila duljine triju stranica istog pravokutnika i dobila 14 cm. Koliki je opseg toga pravokutnika.

$$A.\ 16\ cm \qquad B.\ 18\ cm \qquad C.\ 20\ cm \qquad D.\ 24\ cm$$

Rezultat: C.

Zadatak 103 (Anchy, gimnazija)

Odredi sve uređene trojke cijelih brojeva (a, b, c) za koje vrijedi $a\cdot b+b\cdot c+c\cdot a=a+b+c+a\cdot b\cdot c.$

Rješenje 103

Ponovimo!

$$a=b\ \Rightarrow\ a+c=b+c\ ,\ c\in R \quad , \quad a+b=a\ \Rightarrow\ b=0.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Cijeli brojevi jesu brojevi:

$$\ldots,\ -5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ \ldots$$

Oni čine skup cijelih brojeva koji označavamo slovom Z, a zapisujemo kao

$$Z=\{\ldots,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ \ldots\} \quad \text{ili} \quad Z=\{0,\ 1,\ -1,\ 2,\ -2,\ 3,\ -3,\ \ldots\}.$$

Danu jednakost preoblikujemo kako bismo dobili faktore.

$$a\cdot b+b\cdot c+c\cdot a=a+b+c+a\cdot b\cdot c\ \Rightarrow\ a\cdot b+b\cdot c+c\cdot a-a-b-c-a\cdot b\cdot c=0\ \Rightarrow$$

$$\Rightarrow\ a\cdot b+b\cdot c+c\cdot a-a-b-c-a\cdot b\cdot c=0\ /+1\ \Rightarrow\ 1+a\cdot b+b\cdot c+c\cdot a-a-b-c-a\cdot b\cdot c=1\ \Rightarrow$$

$$\Rightarrow\ 1-a-b-c+a\cdot b+b\cdot c+c\cdot a-a\cdot b\cdot c=1\ \Rightarrow\ \begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix}\ \Rightarrow$$

$$\Rightarrow\ (1-a)+(b\cdot c-a\cdot b\cdot c)+(a\cdot b-b)+(c\cdot a-c)=1\ \Rightarrow$$

$$\Rightarrow\ (1-a)+b\cdot c\cdot(1-a)-b\cdot(1-a)-c\cdot(1-a)=1\ \Rightarrow$$

$$\Rightarrow\ (1-a)+b\cdot c\cdot(1-a)-b\cdot(1-a)-c\cdot(1-a)=1\ \Rightarrow\ (1-a)\cdot(1+b\cdot c-b-c)=1\ \Rightarrow$$

$$\Rightarrow\ (1-a)\cdot((1-b)-c\cdot(1-b))=1\ \Rightarrow\ (1-a)\cdot((1-b)-c\cdot(1-b))=1\ \Rightarrow$$

$$\Rightarrow\ (1-a)\cdot(1-b)\cdot(1-c)=1.$$

Dobili smo jednadžbu

$$(1-a)\cdot(1-b)\cdot(1-c)=1.$$

Nju ćemo riješiti imajući u vidu da su a, b i c cijeli brojevi.

- $$(1-a)\cdot(1-b)\cdot(1-c)=1\ \Rightarrow\ \left.\begin{matrix}1-a=1\\1-b=1\\1-c=1\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix}1-a=1\\1-b=1\\1-c=1\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix}a=0\\b=0\\c=0\end{matrix}\right\}\ \Rightarrow\ (a,\ b,\ c)=(0,\ 0,\ 0).$$

- $$(1-a)\cdot(1-b)\cdot(1-c)=1\ \Rightarrow\ \left.\begin{matrix}1-a=-1\\1-b=-1\\1-c=1\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix}-a=-1-1\\-b=-1-1\\1-c=1\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix}-a=-2\\-b=-2\\c=0\end{matrix}\right\}\ \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -a=-2 \;/\cdot(-1) \\ -b=-2 \;/\cdot(-1) \\ c=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=2 \\ b=2 \\ c=0 \end{array}\right\} \Rightarrow (a,\ b,\ c)=(2,\ 2,\ 0)$$

- $$(1-a)\cdot(1-b)\cdot(1-c)=1 \Rightarrow \left.\begin{array}{l} 1-a=-1 \\ 1-b=1 \\ 1-c=-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -a=-1-1 \\ 1-b=1 \\ -c=-1-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -a=-2 \\ b=0 \\ -c=-2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -a=-2 \;/\cdot(-1) \\ b=0 \\ -c=-2 \;/\cdot(-1) \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=2 \\ b=0 \\ c=2 \end{array}\right\} \Rightarrow (a,\ b,\ c)=(2,\ 0,\ 2)$$

- $$(1-a)\cdot(1-b)\cdot(1-c)=1 \Rightarrow \left.\begin{array}{l} 1-a=1 \\ 1-b=-1 \\ 1-c=-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 1-a=1 \\ -b=-1-1 \\ -c=-1-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=0 \\ -b=-2 \\ -c=-2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a=0 \\ -b=-2 \;/\cdot(-1) \\ -c=-2 \;/\cdot(-1) \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=0 \\ b=2 \\ c=2 \end{array}\right\} \Rightarrow (a,\ b,\ c)=(0,\ 2,\ 2).$$

Vježba 103

Odredi sve uređene trojke cijelih brojeva (a, b, c) za koje vrijedi
$a\cdot b+b\cdot c+c\cdot a-a\cdot b\cdot c=a+b+c.$

Rezultat: $(a,\ b,\ c)=(0,\ 0,\ 0)$, $(a,\ b,\ c)=(2,\ 2,\ 0)$
$(a,\ b,\ c)=(2,\ 0,\ 2)$, $(a,\ b,\ c)=(0,\ 2,\ 2).$

Zadatak 104 (Miroslav, gimnazija)

Koliki je zbroj x + y + z, ako su zadovoljene sljedeće jednadžbe; $\left\{\begin{array}{l} x+2\cdot y+2\cdot z=6 \\ -2\cdot x+y-2\cdot z=-25 \\ 2\cdot x-2\cdot y+z=24 \end{array}\right.$?

$A.\ -5 \qquad B.\ 5 \qquad C.\ -3 \qquad D.\ 3$

Rješenje 104

Ponovimo!

$$\left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow a+c=b+d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left.\begin{array}{l} x+2\cdot y+2\cdot z=6 \\ -2\cdot x+y-2\cdot z=-25 \\ 2\cdot x-2\cdot y+z=24 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow$$

$$\Rightarrow (x+2\cdot y+2\cdot z)+(-2\cdot x+y-2\cdot z)+(2\cdot x-2\cdot y+z)=6+(-25)+24 \Rightarrow$$

$$\Rightarrow x+2\cdot y+2\cdot z-2\cdot x+y-2\cdot z+2\cdot x-2\cdot y+z=6-25+24 \Rightarrow$$

$$\Rightarrow\ x+2\cdot y+2\cdot z-2\cdot x+y-2\cdot z+2\cdot x-2\cdot y+z=6-25+24\ \Rightarrow\ x+y+z=5.$$

Odgovor je pod B.

Vježba 104

Ako je (x, y, z) rješenje sustava $\begin{cases} 2\cdot x+3\cdot y+5\cdot z=10 \\ 3\cdot x+7\cdot y+4\cdot z=3 \\ x+2\cdot y+2\cdot z=3, \end{cases}$ onda $6\cdot x+12\cdot y+11\cdot z$ iznosi:

$$A.\ 4 \qquad B.\ 9 \qquad C.\ 16 \qquad D.\ 25$$

Rezultat: C.

Zadatak 105 (4A, TUPŠ)

Za 120 kn mogle su se kupiti dvije čokolade više nego nakon njihova poskupljenja od 25%.

a) Koliko se čokolada moglo kupiti prije poskupljenja?

b) Kolika je cijena jedne čokolade nakon poskupljenja?

Rješenje 105

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}\cdot\frac{b}{a}=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Stoti dio nekog broja naziva se postotak. Piše se kao razlomak s nazivnikom 100. Postotak p je broj jedinica koji se uzima od 100 jedinica neke veličine.

Na primjer,

$$9\ \%=\frac{9}{100} \quad , \quad 81\ \%=\frac{81}{100} \quad , \quad 4.5\ \%=\frac{4.5}{100} \quad , \quad 547\ \%=\frac{547}{100} \quad , \quad p\ \%=\frac{p}{100}.$$

Kako se računa "... p% od x..."?

$$\frac{p}{100}\cdot x.$$

Kako zapisati da se x poveća za p% ?

$$x+\frac{p}{100}\cdot x.$$

Neka je:

- x – broj čokolada prije poskupljenja
- y – cijena jedne čokolade prije poskupljenja.

Tada vrijedi jednadžba

$$x\cdot y=120.$$

Nakon poskupljenja od 25% mogle su se kupiti dvije čokolade manje za 120 kn pa sada jednadžba glasi:

$$(x-2)\cdot\left(y+\frac{25}{100}\cdot y\right)=120\ \Rightarrow\ (x-2)\cdot\left(y+\frac{25}{100}\cdot y\right)=120\ \Rightarrow\ (x-2)\cdot\left(y+\frac{1}{4}\cdot y\right)=120\ \Rightarrow$$

$$\Rightarrow\ (x-2)\cdot y\cdot\left(1+\frac{1}{4}\right)=120\ \Rightarrow\ (x-2)\cdot y\cdot\left(\frac{1}{1}+\frac{1}{4}\right)=120\ \Rightarrow\ (x-2)\cdot y\cdot\frac{4+1}{4}=120\ \Rightarrow$$

$$\Rightarrow\ (x-2)\cdot y\cdot\frac{5}{4}=120\ \Rightarrow\ \frac{5}{4}\cdot(x-2)\cdot y=120.$$

Sustav jednadžbi

$$\begin{cases} x\cdot y=120 \\ \frac{5}{4}\cdot(x-2)\cdot y=120 \end{cases}$$

možemo riješiti na dva načina.

1.inačica

$$\left.\begin{array}{l} x\cdot y=120 \\ \frac{5}{4}\cdot(x-2)\cdot y=120 \end{array}\right\}\Rightarrow\ \frac{5}{4}\cdot(x-2)\cdot y=x\cdot y\ \Rightarrow\ \frac{5}{4}\cdot(x-2)\cdot y=x\cdot y\ /\cdot\frac{4}{y}\ ,\ \ y\neq 0\ \Rightarrow$$

$$\Rightarrow\ 5\cdot(x-2)=4\cdot x\ \Rightarrow\ 5\cdot x-10=4\cdot x\ \Rightarrow\ 5\cdot x-4\cdot x=10\ \Rightarrow\ x=10.$$

Računamo y.

$$\left.\begin{array}{l} x\cdot y=120 \\ x=10 \end{array}\right\}\Rightarrow\ 10\cdot y=120\ \Rightarrow\ 10\cdot y=120\ /:10\ \Rightarrow\ y=12.$$

2.inačica

$$\left.\begin{array}{l} x\cdot y=120 \\ \frac{5}{4}\cdot(x-2)\cdot y=120 \end{array}\right\}\Rightarrow\ \left.\begin{array}{l} x\cdot y=120 \\ \frac{5}{4}\cdot(x-2)\cdot y=120\ /\cdot\frac{4}{5} \end{array}\right\}\Rightarrow\ \left.\begin{array}{l} x\cdot y=120 \\ (x-2)\cdot y=96 \end{array}\right\}\Rightarrow$$

$$\Rightarrow\ \left.\begin{array}{l} x\cdot y=120 \\ x\cdot y-2\cdot y=96 \end{array}\right\}\Rightarrow\ \begin{bmatrix}\text{metoda} \\ \text{zamjene}\end{bmatrix}\Rightarrow\ 120-2\cdot y=96\ \Rightarrow\ -2\cdot y=96-120\ \Rightarrow$$

$$\Rightarrow\ -2\cdot y=-24\ \Rightarrow\ -2\cdot y=-24\ /:(-2)\ \Rightarrow\ y=12.$$

Računamo x.

$$\left.\begin{array}{l} x\cdot y=120 \\ y=12 \end{array}\right\}\Rightarrow\ 12\cdot x=120\ \Rightarrow\ 12\cdot x=120\ /:12\ \Rightarrow\ x=10.$$

a) Prije poskupljenja moglo se kupiti 10 čokolada.

b) Nakon poskupljenja cijena jedne čokolade je

$$y+\frac{25}{100}\cdot y=12+\frac{25}{100}\cdot 12=12+\frac{25}{100}\cdot 12=12+\frac{1}{4}\cdot 12=12+\frac{1}{4}\cdot 12=12+3=15\ kn.$$

Vježba 105

Za 240 kn mogle su se kupiti dvije čokolade više nego nakon njihova poskupljenja od 25%.
a) Koliko se čokolada moglo kupiti prije poskupljenja?
b) Kolika je cijena jedne čokolade nakon poskupljenja?

Rezultat: 10 čokolada, 30 kn.

Zadatak 106 (4A, TUPŠ)

Cijena C unajmljivanja bungalova na n tjedana dana je formulom $C=t\cdot n+d$ (t je iznos tjednog najma, d je sigurnosni depozit). Martina je za 3 tjedna platila 2092 kn, a Maja za 5 tjedana 3412 kn. Koliki je sigurnosni depozit?

Rješenje 106

Ponovimo!

$$a=b\ \Rightarrow\ b=a.$$

Uvrstimo li zadane uvjete u formulu

$$C=t\cdot n+d,$$

dobije se sustav dviju linearnih jednadžbi s dvjema nepoznanicama.

$$\left.\begin{matrix} C = 2092 \ , \ n = 3 \\ C = 3412 \ , \ n = 5 \end{matrix}\right\} \Rightarrow \left[C = t \cdot n + d \right] \Rightarrow \left.\begin{matrix} 2092 = 3 \cdot t + d \\ 3412 = 5 \cdot t + d \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 3 \cdot t + d = 2092 \\ 5 \cdot t + d = 3412 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{matrix} \text{metoda suprotnih} \\ \text{koeficijenata} \end{matrix}\right] \Rightarrow \left.\begin{matrix} 3 \cdot t + d = 2092 \ / \cdot 5 \\ 5 \cdot t + d = 3412 \ / \cdot (-3) \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 15 \cdot t + 5 \cdot d = 10460 \\ -15 \cdot t - 3 \cdot d = -10236 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow 2 \cdot d = 224 \Rightarrow 2 \cdot d = 224 \ / : 2 \Rightarrow d = 112 \ kn.$$



Vježba 106

Cijena C unajmljivanja bungalova na n tjedana dana je formulom $C = t \cdot n + d$ (t je iznos tjednog najma, d je sigurnosni depozit). Martina je za 6 tjedana platila 4184 kn, a Maja za 10 tjedana 6824 kn. Koliki je sigurnosni depozit?

Rezultat: 112 kn.

Zadatak 107 (Amir, tehnička škola)

Riješi sustav jednadžbi: $\begin{cases} 5 \cdot x + 6 \cdot y - 1 = 0 \\ 3 \cdot x^2 + 2 \cdot x \cdot y + 5 \cdot y^2 - 6 = 0. \end{cases}$

Rješenje 107

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2 \quad , \quad n = \frac{n}{1} \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Iz prve, linearne, jednadžbe izračunamo, na primjer, nepoznanicu y.

$$5 \cdot x + 6 \cdot y - 1 = 0 \Rightarrow 6 \cdot y = 1 - 5 \cdot x \Rightarrow 6 \cdot y = 1 - 5 \cdot x \ / : 6 \Rightarrow y = \frac{1 - 5 \cdot x}{6}.$$

Vrijednost za y uvrstimo u drugu, kvadratnu, jednadžbu.

$$\left.\begin{matrix} y = \frac{1 - 5 \cdot x}{6} \\ 3 \cdot x^2 + 2 \cdot x \cdot y + 5 \cdot y^2 - 6 = 0 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{metoda} \\ \text{zamjene} \end{matrix}\right] \Rightarrow 3 \cdot x^2 + 2 \cdot x \cdot \frac{1 - 5 \cdot x}{6} + 5 \cdot \left(\frac{1 - 5 \cdot x}{6}\right)^2 - 6 = 0 \Rightarrow$$

$$\Rightarrow\ 3\cdot x^2+2\cdot x\cdot\frac{1-5\cdot x}{6}+5\cdot\frac{(1-5\cdot x)^2}{6^2}-6=0\ \Rightarrow$$

$$\Rightarrow\ 3\cdot x^2+x\cdot\frac{1-5\cdot x}{3}+5\cdot\frac{1-10\cdot x+25\cdot x^2}{36}-6=0\ \Rightarrow$$

$$\Rightarrow\ 3\cdot x^2+\frac{x}{1}\cdot\frac{1-5\cdot x}{3}+\frac{5}{1}\cdot\frac{1-10\cdot x+25\cdot x^2}{36}-6=0\ \Rightarrow$$

$$\Rightarrow\ 3\cdot x^2+\frac{x-5\cdot x^2}{3}+\frac{5-50\cdot x+125\cdot x^2}{36}-6=0\ \Rightarrow$$

$$\Rightarrow\ 3\cdot x^2+\frac{x-5\cdot x^2}{3}+\frac{5-50\cdot x+125\cdot x^2}{36}-6=0\ /\cdot 36\ \Rightarrow$$

$$\Rightarrow\ 108\cdot x^2+12\cdot\left(x-5\cdot x^2\right)+5-50\cdot x+125\cdot x^2-216=0\ \Rightarrow$$

$$\Rightarrow\ 108\cdot x^2+12\cdot x-60\cdot x^2+5-50\cdot x+125\cdot x^2-216=0\ \Rightarrow$$

$$\Rightarrow\ 173\cdot x^2-38\cdot x-211=0\ \Rightarrow\ \left.\begin{array}{l}173\cdot x^2-38\cdot x-211=0\\ a=173\ ,\ b=-38\ ,\ c=-211\end{array}\right\}\Rightarrow$$

$$\Rightarrow\ \left.\begin{array}{l}a=173\ ,\ b=-38\ ,\ c=-211\\ x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\}\Rightarrow\ x_{1,2}=\frac{-(-38)\pm\sqrt{(-38)^2-4\cdot 173\cdot(-211)}}{2\cdot 173}\ \Rightarrow$$

$$\Rightarrow\ x_{1,2}=\frac{38\pm\sqrt{1444+146\,012}}{346}\ \Rightarrow\ x_{1,2}=\frac{38\pm\sqrt{1476\,456}}{346}\ \Rightarrow\ x_{1,2}=\frac{38\pm 384}{346}\ \Rightarrow$$

$$\Rightarrow\ \left.\begin{array}{l}x_1=\dfrac{38+384}{346}\\ x_2=\dfrac{38-384}{346}\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}x_1=\dfrac{422}{346}\\ x_2=-\dfrac{346}{346}\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}x_1=\dfrac{422}{346}\\ x_2=-\dfrac{346}{346}\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}x_1=\dfrac{211}{173}\\ x_2=-1\end{array}\right\}.$$

Vraćamo se na zamjenu.

$$\left.\begin{array}{l}x_1=\dfrac{211}{173}\\ x_2=-1\end{array}\right\}\Rightarrow\ \left[\ y=\frac{1-5\cdot x}{6}\ \right]\Rightarrow\ \left.\begin{array}{l}y_1=\dfrac{1-5\cdot\frac{211}{173}}{6}\\ y_2=\dfrac{1-5\cdot(-1)}{6}\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}y_1=\dfrac{1-\frac{5}{1}\cdot\frac{211}{173}}{6}\\ y_2=\dfrac{1+5}{6}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\ \left.\begin{array}{l}y_1=\dfrac{1-\frac{1055}{173}}{6}\\ y_2=\dfrac{6}{6}\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}y_1=\dfrac{\frac{1}{1}-\frac{1055}{173}}{6}\\ y_2=\dfrac{6}{6}\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}y_1=\dfrac{\frac{173-1055}{173}}{6}\\ y_2=1\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}y_1=\dfrac{-\frac{882}{173}}{6}\\ y_2=1\end{array}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} y_1 = \dfrac{-\dfrac{882}{173}}{\dfrac{6}{1}} \\ y_2 = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} y_1 = -\dfrac{882}{1038} \\ y_2 = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} y_1 = -\dfrac{882}{1038} \\ y_2 = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} y_1 = -\dfrac{147}{173} \\ y_2 = 1 \end{array}\right\}.$$

Rješenja sustava glase:

$$(x_1,\ y_1) = \left(\frac{211}{173},\ -\frac{147}{173}\right) \quad , \quad (x_2,\ y_2) = (-1,\ 1).$$

Vježba 107

Riješi sustav jednadžbi: $\begin{cases} 5 \cdot x + 6 \cdot y = 1 \\ 3 \cdot (x^2 - 2) + 2 \cdot x \cdot y + 5 \cdot y^2 = 0. \end{cases}$

Rezultat: $(x_1,\ y_1) = \left(\frac{211}{173},\ -\frac{147}{173}\right) \quad , \quad (x_2,\ y_2) = (-1,\ 1).$

Zadatak 108 (Amir, tehnička škola)

Riješi sustav jednadžbi: $\begin{cases} x^2 + y^2 = 8 \\ x \cdot y = 4. \end{cases}$

Rješenje 108

Ponovimo!

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad (a^n)^m = a^{n \cdot m} \quad , \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2.$$

$$a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2 \quad , \quad a^2 = 0 \Rightarrow a = 0 \quad , \quad \left.\begin{array}{l} a = b \\ c = d \end{array}\right\} \Rightarrow a + c = b + d.$$

$$a^1 = a.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

$$\left.\begin{array}{l} x^2 + y^2 = 8 \\ x \cdot y = 4 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2 + y^2 = 8 \\ x \cdot y = 4 \ / \cdot \frac{1}{x} \ , \ x \neq 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2 + y^2 = 8 \\ y = \frac{4}{x} \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow$$

$$\Rightarrow x^2 + \left(\frac{4}{x}\right)^2 = 8 \Rightarrow x^2 + \frac{4^2}{x^2} = 8 \Rightarrow x^2 + \frac{16}{x^2} = 8 \Rightarrow x^2 + \frac{16}{x^2} = 8 \ / \cdot x^2 \Rightarrow$$

$$\Rightarrow x^4 + 16 = 8 \cdot x^2 \Rightarrow x^4 + 16 - 8 \cdot x^2 = 0 \Rightarrow x^4 - 8 \cdot x^2 + 16 = 0 \Rightarrow$$

$$\Rightarrow (x^2)^2 - 8 \cdot x^2 + 4^2 = 0 \Rightarrow (x^2 - 4)^2 = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow$$

$$\Rightarrow\ x^2=4\ /\sqrt{\ }\ \Rightarrow\ x_{1,2}=\pm\sqrt{4}\ \Rightarrow\ x_{1,2}=\pm 2\ \Rightarrow\ \left.\begin{matrix} x_1=2 \\ x_2=-2 \end{matrix}\right\}.$$

Vraćamo se na zamjenu.

$$\left.\begin{matrix} x_1=2 \\ x_2=-2 \end{matrix}\right\}\ \Rightarrow\ \left[\ y=\frac{4}{x}\ \right]\ \Rightarrow\ \left.\begin{matrix} y_1=\frac{4}{2} \\ y_2=\frac{4}{-2} \end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} y_1=\frac{4}{2} \\ y_2=\frac{4}{-2} \end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} y_1=2 \\ y_2=-2 \end{matrix}\right\}.$$

Rješenja sustava glase:

$$\left(x_1,\ y_1\right)=(2,\ 2)\quad,\quad \left(x_2,\ y_2\right)=(-2,\ -2).$$

2.inačica

$$\left.\begin{matrix} x^2+y^2=8 \\ x\cdot y=4 \end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} x^2+y^2=8 \\ x\cdot y=4\ /\cdot 2 \end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} x^2+y^2=8 \\ 2\cdot x\cdot y=8 \end{matrix}\right\}\ \Rightarrow\ \left[\begin{matrix} \text{zbrojimo} \\ \text{jednadžbe} \end{matrix}\right]\ \Rightarrow$$

$$\Rightarrow\ x^2+y^2+2\cdot x\cdot y=8+8\ \Rightarrow\ x^2+2\cdot x\cdot y+y^2=16\ \Rightarrow\ (x+y)^2=16\ \Rightarrow$$

$$\Rightarrow\ (x+y)^2=16\ /\sqrt{\ }\ \Rightarrow\ x+y=\pm\sqrt{16}\ \Rightarrow\ x+y=\pm 4\ \Rightarrow\ \left.\begin{matrix} x+y=4 \\ x+y=-4 \end{matrix}\right\}.$$

Moramo riješiti dva sustava jednadžbi.

- $$\left.\begin{matrix} x+y=4 \\ x\cdot y=4 \end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} y=4-x \\ x\cdot y=4 \end{matrix}\right\}\ \Rightarrow\ \left[\begin{matrix} \text{metoda} \\ \text{zamjene} \end{matrix}\right]\ \Rightarrow\ x\cdot(4-x)=4\ \Rightarrow\ 4\cdot x-x^2=4\ \Rightarrow$$

$$\Rightarrow\ 4\cdot x-x^2-4=0\ \Rightarrow\ -x^2+4\cdot x-4=0\ \Rightarrow\ -x^2+4\cdot x-4=0\ /\cdot(-1)\ \Rightarrow$$

$$\Rightarrow\ x^2-4\cdot x+4=0\ \Rightarrow\ (x-2)^2=0\ \Rightarrow\ x-2=0\ \Rightarrow\ x_1=2.$$

Računamo y_1.

$$\left.\begin{matrix} x+y=4 \\ x=2 \end{matrix}\right\}\ \Rightarrow\ 2+y=4\ \Rightarrow\ y=4-2\ \Rightarrow\ y_1=2.$$

Rješenje sustava je uređeni par

$$\left(x_1,\ y_1\right)=(2,\ 2).$$

- $$\left.\begin{matrix} x+y=-4 \\ x\cdot y=4 \end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} y=-4-x \\ x\cdot y=4 \end{matrix}\right\}\ \Rightarrow\ \left[\begin{matrix} \text{metoda} \\ \text{zamjene} \end{matrix}\right]\ \Rightarrow\ x\cdot(-4-x)=4\ \Rightarrow\ -4\cdot x-x^2=4\ \Rightarrow$$

$$\Rightarrow\ -4\cdot x-x^2-4=0\ \Rightarrow\ -x^2-4\cdot x-4=0\ \Rightarrow\ -x^2-4\cdot x-4=0\ /\cdot(-1)\ \Rightarrow$$

$$\Rightarrow\ x^2+4\cdot x+4=0\ \Rightarrow\ (x+2)^2=0\ \Rightarrow\ x+2=0\ \Rightarrow\ x_2=-2.$$

Računamo y_2.

$$\left.\begin{matrix} x+y=-4 \\ x=-2 \end{matrix}\right\}\ \Rightarrow\ -2+y=-4\ \Rightarrow\ y=-4+2\ \Rightarrow\ y_2=-2.$$

Rješenje sustava je uređeni par

$$\left(x_2,\ y_2\right)=(-2,\ -2).$$

Vježba 108

Riješi sustav jednadžbi: $\begin{cases} x^2 + y^2 - 8 = 0 \\ x \cdot y - 4 = 0. \end{cases}$

Rezultat: $(x_1, y_1) = (2, 2) \quad , \quad (x_2, y_2) = (-2, -2).$

Zadatak 109 (4A, TUPŠ)

Neka je a zadan realan broj. U sustavu jednadžbi $\begin{cases} 3 \cdot x + 2 \cdot y = a \\ 2 \cdot x + y - 1 = 0. \end{cases}$ odredite nepoznanicu x.

$A.\ 2 - a \qquad B.\ a - 2 \qquad C.\ 2 + a \qquad D.\ a$

Rješenje 109

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$\left.\begin{array}{l} 3 \cdot x + 2 \cdot y = a \\ 2 \cdot x + y - 1 = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3 \cdot x + 2 \cdot y = a \\ 2 \cdot x + y = 1 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 3 \cdot x + 2 \cdot y = a \\ 2 \cdot x + y = 1 \ / \cdot (-2) \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3 \cdot x + 2 \cdot y = a \\ -4 \cdot x - 2 \cdot y = -2 \end{array}\right\} \Rightarrow -x = a - 2 \Rightarrow$$

$$\Rightarrow -x = a - 2 \ / \cdot (-1) \Rightarrow x = -a + 2 \Rightarrow x = 2 - a.$$

Odgovor je pod A.

Vježba 109

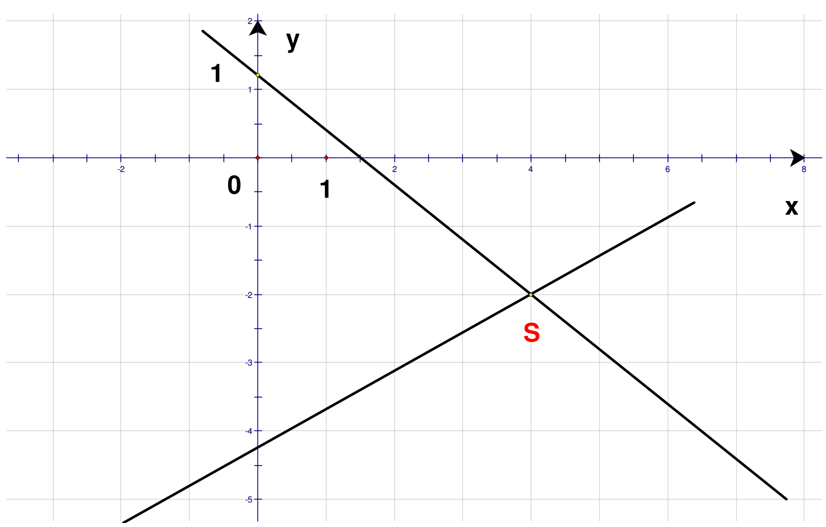
Neka je a zadan realan broj. U sustavu jednadžbi $\begin{cases} 3 \cdot x + 2 \cdot y = a \\ 2 \cdot x + y - 1 = 0. \end{cases}$ odredite nepoznanicu y.

$A.\ 2 + 3 \cdot a \qquad B.\ 2 \cdot a - 2 \qquad C.\ 2 \cdot a - 3 \qquad D.\ 3 \cdot a + 2$

Rezultat: C.

Zadatak 110 (4A, 4B, TUPŠ)

Sustav jednadžbi $\begin{cases} a \cdot x - y + 1 = 0 \\ 3 \cdot x - 8 \cdot y + b = 0 \end{cases}$ riješen je grafički.



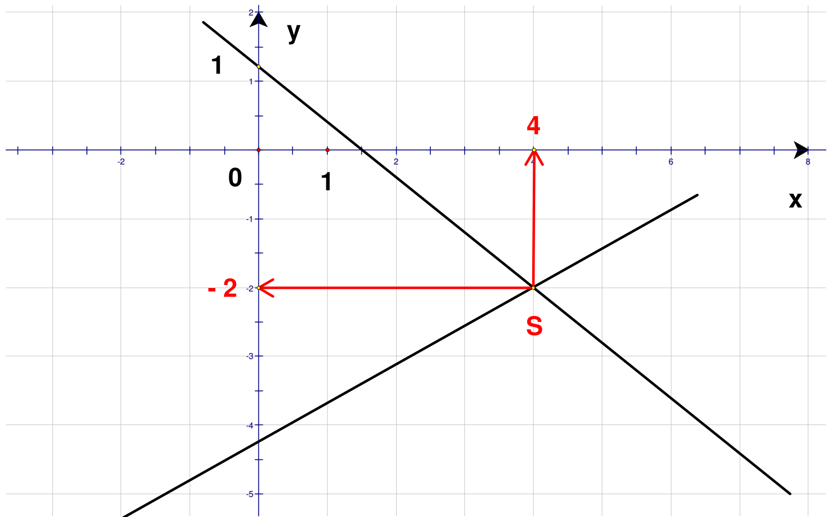
Odredite realne brojeve a i b.

Rješenje 110

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$



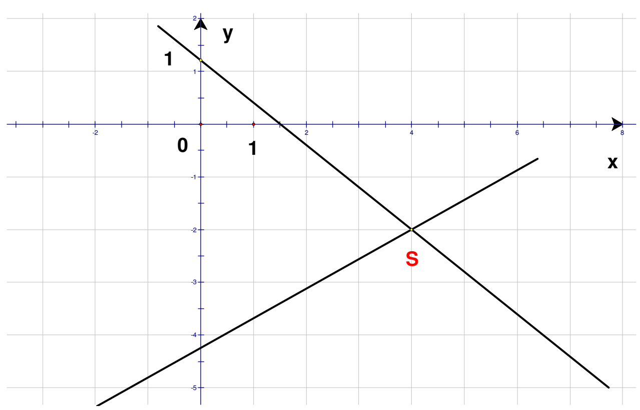
Sa slike vidi se da je sjecište pravaca točka S(4, – 2) . Budući da ona pripada i jednom i drugom pravcu, uvrstit ćemo njezine koordinate u sustav jednadžbi i izračunati nepoznanice a i b.

$$\left.\begin{array}{l} S(x, y) = S(4, -2) \\ a \cdot x - y + 1 = 0 \\ 3 \cdot x - 8 \cdot y + b = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a \cdot 4 - (-2) + 1 = 0 \\ 3 \cdot 4 - 8 \cdot (-2) + b = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 4 \cdot a + 2 + 1 = 0 \\ 12 + 16 + b = 0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 4 \cdot a = -2 - 1 \\ b = -12 - 16 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 4 \cdot a = -3 \\ b = -28 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 4 \cdot a = -3 \ /:4 \\ b = -28 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = -\frac{3}{4} \\ b = -28 \end{array}\right\}.$$

Vježba 110

Sustav jednadžbi $\left\{\begin{array}{l} a \cdot x - y - 22 = 0 \\ 3 \cdot x - 8 \cdot y + b = 0 \end{array}\right.$ riješen je grafički.



Odredite realne brojeve a i b.

Rezultat: $a = 5 \ , \ b = -28.$

Zadatak 111 (Lana, gimnazija)

U košari je 89 kuglica – neke su male, a neke velike. Svaka mala kuglica ima masu 2 g, a svaka velika 5 g. Ukupna masa kuglica u košari je 256 g. Koliko je malih kuglica u košari?

Rješenje 111

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

Neka je:

- x broj malih kuglica
- y broj velikih kuglica.

U košari ukupno je 89 kuglica pa vrijedi jednadžba

$$x + y = 89.$$

Ako mala kuglica ima masu 2 g, velika 5 g, a ukupna masa kuglica u košari je 256 g možemo napisati jednadžbu

$$2 \cdot x + 5 \cdot y = 256.$$

Iz sustava jednadžbi izračunamo x broj malih kuglica.

$$\left.\begin{array}{l} x + y = 89 \\ 2 \cdot x + 5 \cdot y = 256 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow \left.\begin{array}{l} x + y = 89 \;/\cdot(-5) \\ 2 \cdot x + 5 \cdot y = 256 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -5 \cdot x - 5 \cdot y = -445 \\ 2 \cdot x + 5 \cdot y = 256 \end{array}\right\} \Rightarrow$$

$$\Rightarrow -3 \cdot x = -189 \Rightarrow -3 \cdot x = -189 \;/:(-3) \Rightarrow x = 63.$$



2.inačica

U košari ukupno je 89 kuglica. Ako slovom x označimo broj malih kuglica, onda velikih kuglica ima 89 – x.

Budući da mala kuglica ima masu 2 g, velika 5 g, a ukupna masa kuglica u košari iznosi 256 g, vrijedi jednadžba:

$$2 \cdot x + 5 \cdot (89 - x) = 256 \Rightarrow 2 \cdot x + 445 - 5 \cdot x = 256 \Rightarrow 2 \cdot x - 5 \cdot x = 256 - 445 \Rightarrow$$

$$\Rightarrow -3 \cdot x = -189 \Rightarrow -3 \cdot x = -189 \;/:(-3) \Rightarrow x = 63.$$

Vježba 111

U košari je 89 kuglica – neke su male, a neke velike. Svaka mala kuglica ima masu 2 g, a svaka velika 5 g. Ukupna masa kuglica u košari je 256 g. Koliko je velikih kuglica u košari?

Rezultat: 26.

Zadatak 112 (Mario, srednja škola)

Riješi sustav jednadžbi $\begin{cases} 4 \cdot y - x = 6 \\ 2 \cdot y^2 - 3 \cdot x = 0. \end{cases}$

Rješenje 112

Ponovimo!

$$a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2 \quad , \quad a^2 = 0 \Rightarrow a = 0.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Sustav linearne i kvadratne jednadžbe rješavamo metodom zamjene (supstitucije).

$$\left.\begin{array}{l}4\cdot y-x=6\\2\cdot y^{2}-3\cdot x=0\end{array}\right\}.$$

Iz linearne jednadžbe izrazimo jednu nepoznanicu, na primjer x.

$$4\cdot y-x=6\ \Rightarrow\ -x=6-4\cdot y\ \Rightarrow\ -x=6-4\cdot y\ /\cdot(-1)\ \Rightarrow\ x=-6+4\cdot y\ \Rightarrow\ x=4\cdot y-6.$$

Nađeni iznos za x uvrstimo u kvadratnu jednadžbu i riješimo je po drugoj nepoznanici y.

$$\left.\begin{array}{l}x=4\cdot y-6\\2\cdot y^{2}-3\cdot x=0\end{array}\right\}\Rightarrow\ 2\cdot y^{2}-3\cdot(4\cdot y-6)=0\ \Rightarrow\ 2\cdot y^{2}-12\cdot y+18=0\ \Rightarrow$$

$$\Rightarrow\ 2\cdot y^{2}-12\cdot y+18=0\ /:2\ \Rightarrow\ y^{2}-6\cdot y+9=0\ \Rightarrow\ (y-3)^{2}=0\ \Rightarrow\ y-3=0\ \Rightarrow\ y=3.$$

Izračunatu vrijednost druge nepoznanice y = 3 uvrstimo u nađeni izraz za prvu nepoznanicu x i izračunamo odgovarajuću vrijednost.

$$\left.\begin{array}{l}y=3\\x=4\cdot y-6\end{array}\right\}\Rightarrow\ x=4\cdot 3-6\ \Rightarrow\ x=12-6\ \Rightarrow\ x=6.$$

Sustav ima jedno rješenje:

$$(x,\ y)=(6,\ 3).$$

Vježba 112

Riješi sustav jednadžbi $\left\{\begin{array}{l}x-4\cdot y+6=0\\3\cdot x-2\cdot y^{2}=0.\end{array}\right.$

Rezultat: $(x,\ y)=(6,\ 3).$

Zadatak 113 (4A, 4B, TUPŠ)

Riješite sustav jednadžba $\left\{\begin{array}{l}\log(3\cdot x+z)=1\\5^{x-y}=0.04\\y+3\cdot z=0\end{array}\right.$.

Rješenje 113

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_{b} a=c \qquad \log_{b} a=b^{c} \qquad a=b^{c}$$
$$\rightarrow$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log *x* zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x=\log x.$$

$$\log 10=1 \quad , \quad \log f(x)=\log g(x)\ \Rightarrow\ f(x)=g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Decimalni broj piše se u obliku decimalnog razlomka tako da se u brojnik napiše zadani decimalni broj bez decimalne točke, a u nazivnik se napiše dekadska jedinica (10, 100, 1000, 10000, 100000, …) koja ima toliko nula koliko decimalni broj ima decimala (znamenaka na decimalnom mjestu, tj. iza

decimalne točke ili decimalnog zareza).

$$a^{-n}=\frac{1}{a^{n}}\quad,\quad a^{f(x)}=a^{g(x)}\ \Rightarrow\ f(x)=g(x).$$

$$\left.\begin{matrix}a=b\\c=d\end{matrix}\right\}\ \Rightarrow\ a+c=b+d.$$

$$\left.\begin{matrix}\log(3\cdot x+z)=1\\5^{x-y}=0.04\\y+3\cdot z=0\end{matrix}\right\}\Rightarrow\left.\begin{matrix}\log(3\cdot x+z)=\log 10\\5^{x-y}=\frac{4}{100}\\y+3\cdot z=0\end{matrix}\right\}\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\5^{x-y}=\frac{4}{100}\\y+3\cdot z=0\end{matrix}\right\}\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\5^{x-y}=\frac{1}{25}\\y+3\cdot z=0\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\5^{x-y}=\frac{1}{5^{2}}\\y+3\cdot z=0\end{matrix}\right\}\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\5^{x-y}=5^{-2}\\y+3\cdot z=0\end{matrix}\right\}\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\x-y=-2\\y+3\cdot z=0\end{matrix}\right\}\Rightarrow\left[\begin{matrix}\text{zbrojimo}\\\text{drugu i treću}\\\text{jednadžbu}\end{matrix}\right]\Rightarrow$$

$$\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\x-y+y+3\cdot z=-2+0\end{matrix}\right\}\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\x-y+y+3\cdot z=-2\end{matrix}\right\}\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\x+3\cdot z=-2\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\left[\begin{matrix}\text{metoda suprotnih}\\\text{koeficijenata}\end{matrix}\right]\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\x+3\cdot z=-2\ /\cdot(-3)\end{matrix}\right\}\Rightarrow\left.\begin{matrix}3\cdot x+z=10\\-3\cdot x-9\cdot z=6\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow -8\cdot z=16\ \Rightarrow\ -8\cdot z=16\ /:(-8)\ \Rightarrow\ z=-2.$$

Računamo x i y.

$$\left.\begin{matrix}3\cdot x+z=10\\y+3\cdot z=0\end{matrix}\right\}\Rightarrow\left[z=-2\right]\Rightarrow\left.\begin{matrix}3\cdot x-2=10\\y+3\cdot(-2)=0\end{matrix}\right\}\Rightarrow\left.\begin{matrix}3\cdot x=10+2\\y-6=0\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{matrix}3\cdot x=12\\y=6\end{matrix}\right\}\Rightarrow\left.\begin{matrix}3\cdot x=12\ /:3\\y=6\end{matrix}\right\}\Rightarrow\left.\begin{matrix}x=4\\y=6\end{matrix}\right\}.$$

Rješenje sustava glasi:

$$(x,\ y,\ z)=(4,\ 6,\ -2).$$

Vježba 113

Riješite sustav jednadžba $\begin{cases}\log(3\cdot x+z)=1\\5^{x-y+1}=0.2\\y+3\cdot z=0\end{cases}$.

Rezultat: $(x,\ y,\ z)=(4,\ 6,\ -2).$

Zadatak 114 (Roby, ekonomska škola)

Odredite y u rješenju sustava $\begin{cases}\sqrt{x+y}=3\\\frac{x}{y}-k=0\end{cases}$.

Rješenje 114

Ponovimo!

$$\left(\sqrt{a}\right)^2 = a.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Svaki broj oblika $\frac{a}{b}$ gdje su a i b cijeli brojevi, $b \neq 0$, naziva se racionalni broj.

$$\left.\begin{matrix} \sqrt{x+y} = 3 \\ \frac{x}{y} - k = 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \sqrt{x+y} = 3 \ /^2 \\ \frac{x}{y} = k \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \left(\sqrt{x+y}\right)^2 = 3^2 \\ \frac{x}{y} = k \ / \cdot y \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x+y = 9 \\ x = k \cdot y \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{metoda} \\ \text{zamjene} \end{bmatrix} \Rightarrow k \cdot y + y = 9 \Rightarrow (k+1) \cdot y = 9 \Rightarrow (k+1) \cdot y = 9 \ / \cdot \frac{1}{k+1} \Rightarrow$$

$$\Rightarrow y = \frac{9}{k+1}, \ k \neq -1.$$

Vježba 114

Odredite y u rješenju sustava $\left\{\begin{matrix} \sqrt{x+y} = 4 \\ \frac{x}{y} - k = 0 \end{matrix}\right.$.

Rezultat: $y = \frac{16}{k+1}, \ k \neq -1.$

Zadatak 115 (4A, 4B, TUPŠ)

I umnožak i količnik dvaju brojeva jednaki su 1. Ako jednog od tih brojeva oduzmemo od drugo, dobit ćemo:

$A. -1 \qquad B. \ 0 \qquad C. \ 1 \qquad D. \ 2$

Rješenje 115

Ponovimo!

$$\frac{a}{b} = 1 \Rightarrow a = b \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a = b \Rightarrow a - b = 0.$$

Neka su x i y traženi brojevi. Prema uvjetima zadatka vrijedi:

$$\left.\begin{matrix} x \cdot y = 1 \\ \frac{x}{y} = 1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x \cdot y = 1 \\ x = y \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{zamjene} \end{bmatrix} \Rightarrow y \cdot y = 1 \Rightarrow y^2 = 1 \Rightarrow y^2 = 1 \ / \sqrt{\ } \Rightarrow$$

$$\Rightarrow y_{1,2} = \pm\sqrt{1} \Rightarrow y_{1,2} = \pm 1 \Rightarrow \left.\begin{matrix} y_1 = 1 \\ y_2 = -1 \end{matrix}\right\}.$$

Računamo x.

$$\left.\begin{matrix} y_1 = 1 \\ y_2 = -1 \end{matrix}\right\} \Rightarrow [\, x = y \,] \Rightarrow \left.\begin{matrix} x_1 = 1 \\ x_2 = -1 \end{matrix}\right\}.$$

Ako jednog od tih brojeva oduzmemo od drugog, dobit ćemo 0.

$$x_1 - y_1 = 1 - 1 = 0 \quad , \quad x_2 - y_2 = -1 - (-1) = -1 + 1 = 0.$$

Odgovor je pod B.

Vježba 115

I zbroj i umnožak dvaju brojeva jednaki su 4. Ako jednog od tih brojeva podijelimo drugim, dobit ćemo:

$$A.\ -1 \qquad B.\ 0 \qquad C.\ 1 \qquad D.\ 2$$

Rezultat: C.

Zadatak 116 (Marko, gimnazija)

Ako x i y zadovoljavaju sustav $2^x \cdot (x+y) = 10$, $\sqrt[x]{x+y} = 5$, tada y iznosi:

$$A.\ 1 \qquad B.\ 3 \qquad C.\ 4 \qquad D.\ 9$$

Rješenje 116

Ponovimo!

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad , \quad \sqrt[n]{a^n} = a \quad , \quad a^1 = a \quad , \quad \sqrt[n]{a^m} = a^{\frac{m}{n}} \quad , \quad \frac{a}{b} = 1 \Rightarrow a = b.$$

$$a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\left.\begin{array}{l} 2^x \cdot (x+y) = 10 \\ \sqrt[x]{x+y} = 5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 2^x \cdot (x+y) = 10 \ / \cdot \frac{1}{2^x} \\ \sqrt[x]{x+y} = 5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x+y = \frac{10}{2^x} \\ \sqrt[x]{x+y} = 5 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow$$

$$\Rightarrow \sqrt[x]{\frac{10}{2^x}} = 5 \Rightarrow \frac{\sqrt[x]{10}}{\sqrt[x]{2^x}} = 5 \Rightarrow \frac{\sqrt[x]{10}}{2} = 5 \Rightarrow \frac{\sqrt[x]{10}}{2} = 5 \ / \cdot 2 \Rightarrow \sqrt[x]{10} = 10 \Rightarrow$$

$$\Rightarrow 10^{\frac{1}{x}} = 10^1 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1.$$

Računamo y.

$$\left.\begin{array}{l} x+y = \frac{10}{2^x} \\ x = 1 \end{array}\right\} \Rightarrow 1+y = \frac{10}{2^1} \Rightarrow 1+y = \frac{10}{2} \Rightarrow 1+y = \frac{10}{2} \Rightarrow 1+y = 5 \Rightarrow y = 5-1 \Rightarrow y = 4.$$

Odgovor je pod C.

Vježba 116

Ako x i y zadovoljavaju sustav $2^x \cdot (x+y) - 10 = 0$, $\sqrt[x]{x+y} - 5 = 0$, tada y iznosi:

$$A.\ 1 \qquad B.\ 3 \qquad C.\ 4 \qquad D.\ 9$$

Rezultat: C.

Zadatak 117 (Toni, gimnazija)

Temperatura T(t) izražena u °C mijenja se prema formuli $T(t) = A \cdot \cos(B \cdot t + C) + D$, gdje je t vrijeme u satima. Kolike su vrijednosti parametara A i D ako je maksimalna temperatura 29 °C, minimalna 13 °C i A < 0?

$$A.\ A = -16 \ , \ D = 21 \qquad B.\ A = -16 \ , \ D = 45$$
$$C.\ A = -8 \ , \ D = 21 \qquad D.\ A = -8 \ , \ D = 45$$

Rješenje 117

Ponovimo!

Funkcija f(x) = cos x ima:

- maksimalnu vrijednost 1
- minimalni vrijednost – 1.

Budući da je parametar A negativan broj (A < 0), temperatura

$$T(t)=A\cdot\cos(B\cdot t+C)+D$$

bit će:

- maksimalna, 29 °C, za $\cos(B\cdot t+C)=-1$

$$A\cdot(-1)+D=29 \;\Rightarrow\; -A+D=29$$

- minimalna, 13 °C, za $\cos(B\cdot t+C)=1$

$$A\cdot 1+D=29 \;\Rightarrow\; A+D=13.$$

Iz sustava jednadžbi izračunamo A i D.

$$\left.\begin{array}{l}-A+D=29\\ A+D=13\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda suprotnih}\\ \text{koeficijenata}\end{array}\right] \Rightarrow 2\cdot D=42 \;\Rightarrow\; 2\cdot D=42 \;/:2 \;\Rightarrow\; D=21.$$

Računamo A.

$$\left.\begin{array}{l}A+D=13\\ D=21\end{array}\right\} \Rightarrow A+21=13 \;\Rightarrow\; A=13-21 \;\Rightarrow\; A=-8.$$

Odgovor je pod C.

Vježba 117

Temperatura T(t) izražena u °C mijenja se prema formuli $T(t)=A\cdot\cos(B\cdot t+C)+D$, gdje je t vrijeme u satima. Kolike su vrijednosti parametara A i D ako je maksimalna temperatura 29 °C, minimalna 13 °C i A > 0?

$A.\ A=16\ ,\ D=21$ $\quad B.\ A=16\ ,\ D=45$

$C.\ A=8\ ,\ D=21$ $\quad D.\ A=8\ ,\ D=45$

Rezultat: C.

Zadatak 118 (Margareta, srednja škola)

Riješi sustav: $\dfrac{x\cdot y}{x+y}=\dfrac{1}{5}\ ,\ \dfrac{x\cdot z}{x+z}=\dfrac{1}{6}\ ,\ \dfrac{y\cdot z}{y+z}=\dfrac{1}{7}.$

Rješenje 118

Ponovimo!

$$\frac{a}{b}=\frac{c}{d} \Rightarrow \frac{b}{a}=\frac{d}{c} \quad , \quad \frac{n}{1}=n \quad , \quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n} \quad , \quad \left.\begin{array}{l}a=b\\ c=d\end{array}\right\} \Rightarrow a+c=b+d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left.\begin{array}{l}\frac{x\cdot y}{x+y}=\frac{1}{5}\\ \frac{x\cdot z}{x+z}=\frac{1}{6}\\ \frac{y\cdot z}{y+z}=\frac{1}{7}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{x+y}{x\cdot y}=5\\ \frac{x+z}{x\cdot z}=6\\ \frac{y+z}{y\cdot z}=7\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{x}{x\cdot y}+\frac{y}{x\cdot y}=5\\ \frac{x}{x\cdot z}+\frac{z}{x\cdot z}=6\\ \frac{y}{y\cdot z}+\frac{z}{y\cdot z}=7\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{x}{x\cdot y}+\frac{y}{x\cdot y}=5\\ \frac{x}{x\cdot z}+\frac{z}{x\cdot z}=6\\ \frac{y}{y\cdot z}+\frac{z}{y\cdot z}=7\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{1}{y}+\frac{1}{x}=5\\ \frac{1}{z}+\frac{1}{x}=6\\ \frac{1}{z}+\frac{1}{y}=7\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left[\begin{array}{c}\text{zbrojimo}\\ \text{jednadžbe}\end{array}\right]\Rightarrow\frac{1}{y}+\frac{1}{x}+\frac{1}{z}+\frac{1}{x}+\frac{1}{z}+\frac{1}{y}=5+6+7\Rightarrow\frac{2}{x}+\frac{2}{y}+\frac{2}{z}=18\Rightarrow$$

$$\Rightarrow 2\cdot\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=18\Rightarrow 2\cdot\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=18\ /:2\Rightarrow\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9.$$

Računamo nepoznanicu x.

$$\left.\begin{array}{l}\frac{1}{z}+\frac{1}{y}=7\\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{1}{y}+\frac{1}{z}=7\\ \frac{1}{x}+\left(\frac{1}{y}+\frac{1}{z}\right)=9\end{array}\right\}\Rightarrow\frac{1}{x}+7=9\Rightarrow\frac{1}{x}=9-7\Rightarrow\frac{1}{x}=2\Rightarrow$$

$$\Rightarrow\frac{1}{x}=\frac{2}{1}\Rightarrow x=\frac{1}{2}.$$

Računamo nepoznanicu y.

$$\left.\begin{array}{l}\frac{1}{x}=2\\ \frac{1}{y}+\frac{1}{x}=5\end{array}\right\}\Rightarrow\frac{1}{y}+2=5\Rightarrow\frac{1}{y}=5-2\Rightarrow\frac{1}{y}=3\Rightarrow\frac{1}{y}=\frac{3}{1}\Rightarrow y=\frac{1}{3}.$$

Računamo nepoznanicu z.

$$\left.\begin{array}{l}\frac{1}{x}=2\\ \frac{1}{z}+\frac{1}{x}=6\end{array}\right\}\Rightarrow\frac{1}{z}+2=6\Rightarrow\frac{1}{z}=6-2\Rightarrow\frac{1}{z}=4\Rightarrow\frac{1}{z}=\frac{4}{1}\Rightarrow z=\frac{1}{4}.$$

Rješenje sustava je uređena trojka:

$$(x,\ y,\ z)=\left(\frac{1}{2},\frac{1}{3},\frac{1}{4}\right).$$

Vježba 118

Riješi sustav: $x+y=5\cdot x\cdot y\ ,\ x+z=6\cdot x\cdot z\ ,\ y+z=7\cdot y\cdot z.$

Rezultat: $(x,\ y,\ z)=\left(\frac{1}{2},\frac{1}{3},\frac{1}{4}\right).$

Zadatak 119 (Tomislav, gimnazija)

Ne rješavajući sustav $\left\{\begin{array}{l}x+y+z=a\\ x^2+y^2+z^2=b^2\\ x^3+y^3+z^3=c^3\end{array}\right.$ nađi $x\cdot y\cdot z.$

Rješenje 119

Ponovimo!

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$(x + y + z)^3 = x^3 + y^3 + z^3 + 3 \cdot \left(x^2 \cdot (y + z) + y^2 \cdot (z + x) + z^2 \cdot (x + y) + 2 \cdot x \cdot y \cdot z\right).$$

Iz prve jednadžbe x + y + z = a dobije se:

- $y + z = a - x$
- $z + x = a - y$
- $x + y = a - z.$

U identitetu

$$(x + y + z)^3 = x^3 + y^3 + z^3 + 3 \cdot \left(x^2 \cdot (y + z) + y^2 \cdot (z + x) + z^2 \cdot (x + y) + 2 \cdot x \cdot y \cdot z\right)$$

napravimo zamjene.

$$(x + y + z)^3 = x^3 + y^3 + z^3 + 3 \cdot \left(x^2 \cdot (y + z) + y^2 \cdot (z + x) + z^2 \cdot (x + y) + 2 \cdot x \cdot y \cdot z\right) \Rightarrow$$

$$\Rightarrow \left[\begin{matrix} \text{zamjene} \\ x + y + z = a \;,\; x^3 + y^3 + z^3 = c^3 \\ y + z = a - x \;,\; z + x = a - y \;,\; x + y = a - z \end{matrix}\right] \Rightarrow$$

$$\Rightarrow a^3 = c^3 + 3 \cdot \left(x^2 \cdot (a - x) + y^2 \cdot (a - y) + z^2 \cdot (a - z) + 2 \cdot x \cdot y \cdot z\right) \Rightarrow$$

$$\Rightarrow a^3 = c^3 + 3 \cdot \left(x^2 \cdot a - x^3 + y^2 \cdot a - y^3 + z^2 \cdot a - z^3 + 2 \cdot x \cdot y \cdot z\right) \Rightarrow$$

$$\Rightarrow a^3 = c^3 + 3 \cdot \left(x^2 \cdot a + y^2 \cdot a + z^2 \cdot a - x^3 - y^3 - z^3 + 2 \cdot x \cdot y \cdot z\right) \Rightarrow$$

$$\Rightarrow a^3 = c^3 + 3 \cdot \left(a \cdot \left(x^2 + y^2 + z^2\right) - \left(x^3 + y^3 + z^3\right) + 2 \cdot x \cdot y \cdot z\right) \Rightarrow$$

$$\Rightarrow \left[\begin{matrix} \text{zamjene} \\ x^2 + y^2 + z^2 = b^2 \;,\; x^3 + y^3 + z^3 = c^3 \end{matrix}\right] \Rightarrow$$

$$\Rightarrow a^3 = c^3 + 3 \cdot \left(a \cdot b^2 - c^3 + 2 \cdot x \cdot y \cdot z\right) \Rightarrow a^3 = c^3 + 3 \cdot a \cdot b^2 - 3 \cdot c^3 + 6 \cdot x \cdot y \cdot z \Rightarrow$$

$$\Rightarrow c^3 + 3 \cdot a \cdot b^2 - 3 \cdot c^3 + 6 \cdot x \cdot y \cdot z = a^3 \Rightarrow 6 \cdot x \cdot y \cdot z = a^3 - c^3 - 3 \cdot a \cdot b^2 + 3 \cdot c^3 \Rightarrow$$

$$\Rightarrow 6 \cdot x \cdot y \cdot z = a^3 - 3 \cdot a \cdot b^2 + 2 \cdot c^3 \Rightarrow 6 \cdot x \cdot y \cdot z = a^3 - 3 \cdot a \cdot b^2 + 2 \cdot c^3 \quad /:6 \Rightarrow$$

$$\Rightarrow x \cdot y \cdot z = \frac{a^3 - 3 \cdot a \cdot b^2 + 2 \cdot c^3}{6}.$$

Vježba 119

Ne rješavajući sustav $\begin{cases} x + y + z = a \\ x^2 + y^2 + z^2 = b^2 \\ x^3 + y^3 + z^3 = c^3 \end{cases}$ nađi $6 \cdot x \cdot y \cdot z.$

Rezultat: $a^3 - 3 \cdot a \cdot b^2 + 2 \cdot c^3.$

Zadatak 120 (Ante, gimnazija)

Riješite sustav $\begin{cases} x \cdot (x+y+z) = 6 \\ y \cdot (x+y+z) = 12 \\ z \cdot (x+y+z) = 18 \end{cases}.$

Rješenje 120

Ponovimo!

$$\left.\begin{array}{l} a = b \\ c = d \end{array}\right\} \Rightarrow a + c = b + d \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$\left.\begin{array}{l} x \cdot (x+y+z) = 6 \\ y \cdot (x+y+z) = 12 \\ z \cdot (x+y+z) = 18 \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow$$

$$\Rightarrow x \cdot (x+y+z) + y \cdot (x+y+z) + z \cdot (x+y+z) = 6 + 12 + 18 \Rightarrow$$

$$\Rightarrow (x+y+z) \cdot (x+y+z) = 36 \Rightarrow (x+y+z)^2 = 36 \Rightarrow (x+y+z)^2 = 36 \; / \sqrt{} \Rightarrow$$

$$\Rightarrow x + y + z = \pm\sqrt{36} \Rightarrow x + y + z = \pm 6 \Rightarrow \left.\begin{array}{l} x + y + z = 6 \\ x + y + z = -6 \end{array}\right\}.$$

Računamo prvi skup rješenja.

$$\left.\begin{array}{l} x \cdot (x+y+z) = 6 \\ y \cdot (x+y+z) = 12 \\ z \cdot (x+y+z) = 18 \end{array}\right\} \Rightarrow [x + y + z = 6] \Rightarrow \left.\begin{array}{r} x \cdot 6 = 6 \\ y \cdot 6 = 12 \\ z \cdot 6 = 18 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 6 \cdot x = 6 \\ 6 \cdot y = 12 \\ 6 \cdot z = 18 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 6 \cdot x = 6 \; /:6 \\ 6 \cdot y = 12 \; /:6 \\ 6 \cdot z = 18 \; /:6 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = 1 \\ y_1 = 2 \\ z_1 = 3 \end{array}\right\}.$$

Računamo drugi skup rješenja.

$$\left.\begin{array}{l} x \cdot (x+y+z) = 6 \\ y \cdot (x+y+z) = 12 \\ z \cdot (x+y+z) = 18 \end{array}\right\} \Rightarrow [x + y + z = -6] \Rightarrow \left.\begin{array}{r} x \cdot (-6) = 6 \\ y \cdot (-6) = 12 \\ z \cdot (-6) = 18 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -6 \cdot x = 6 \\ -6 \cdot y = 12 \\ -6 \cdot z = 18 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -6 \cdot x = 6 \; /:(-6) \\ -6 \cdot y = 12 \; /:(-6) \\ -6 \cdot z = 18 \; /:(-6) \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_2 = -1 \\ y_2 = -2 \\ z_3 = -3 \end{array}\right\}.$$

Rješenja sustava glase:

$$(x_1, y_1, z_1) = (1, 2, 3) \quad , \quad (x_2, y_2, z_2) = (-1, -2, -3).$$

Vježba 120

Riješite sustav $\begin{cases} z \cdot (x + y + z) = 6 \\ y \cdot (x + y + z) = 12 \\ x \cdot (x + y + z) = 18 \end{cases}$.

Rezultat: $(x_1, y_1, z_1) = (3, 2, 1)$, $(x_2, y_2, z_2) = (-3, -2, -1)$.